Benha University **Faculty of Applied Arts** **Mathematics 1 Code : FW 1106**	كلية الفنون التطبيقية جامعة بنها	**Final Term Exam** **Date:10th of January 2018** **Duration : 2 hours**

- Answer all the following questions
- The exam. Consists of one page
- No. of questions: 3
- Total marks: 60

Question 1 [20]

a) $$f(x) = \begin{cases} Ax - B & x \le -1 \\ 2x^2 + 3Ax + B & -1 \le x \le 1 \\ 4 & x > 1 \end{cases}$$

Determine A and B so that the function f (x) is continuous for all values of x

b) Given $\ell n(u) + y\cos u = \arccos(y^2)$ and $u\,e^{3x} + 3x = u^2$, find $\frac{dy}{dx}$.

Question 2 [20]

a) Find the tangent and normal lines of the curve $x^{\cot x}$ at $x = \frac{\pi}{2}$

b) Evaluate the following limits a) $\lim\limits_{x\to 0} [\frac{1}{\sin x} - \frac{1}{x}]$, b) $\lim\limits_{x\to 0} \tan x \,\ell n x$

Question 3 [20]

a) Expand $f(x) = x^3\ \ell n(x+1)$ using Taylor about x = 1

b) Evaluate n^{th} derivative of the following functions

a) $f(x) = \cos(5x)\sin^2(2x)$, b) $g(x) = \ell n[\sqrt[7]{\frac{12 - x - x^2}{25 - x^2}}]$

Good Luck Dr. eng. Khaled el Naggar

Model answer

Answer of Q1

a) Since f (x) is continuous for all values of x, therefore f (x) is continuous at x = -1 and at x = 1.

$$\lim_{x\to -1^+} 2x^2 + 3Ax + B = \lim_{x\to -1^-} Ax - B \Rightarrow 2 - 3A + B = -A - B \Rightarrow A - B = 1$$

$$\lim_{x\to 1^-} 2x^2 + 3Ax + B = \lim_{x\to 1^+} 4 \Rightarrow 2 + 3A + B = 4 \Rightarrow 3A + B = 2$$, hence 4A = 3 $\Rightarrow$

$A = \frac{3}{4}$, $B = -\frac{1}{4}$.

b) $\frac{1}{u} + y(-\sin u) + \cos u \frac{dy}{du} = \frac{-2y}{\sqrt{1-y^4}} \frac{dy}{du} \Rightarrow [\cos u + \frac{2y}{\sqrt{1-y^4}}] \frac{dy}{du} = y \sin u - \frac{1}{u}$

$$\Rightarrow \frac{dy}{du} = \frac{[y \sin u - \frac{1}{u}]}{[\cos u + \frac{2y}{\sqrt{1-y^4}}]}.$$

And $u\,(3e^{3x}) + e^{3x}\frac{du}{dx} + 3 = 2u\frac{du}{dx} \Rightarrow 3u\,e^{3x} + 3 = [2u - e^{3x}]\,\frac{du}{dx} \Rightarrow$

$$\frac{du}{dx} = \frac{3u\,e^{3x} + 3}{2u - e^{3x}}$$

Answer of Q2

a) $y` = x^{\cot x}\,[-\csc^2 x\ \ell n\, x + \frac{1}{x}\cot x]$, thus the slope of the tangent is $y`(\frac{\pi}{2}) = -\ell n\,\frac{\pi}{2}$

Therefore the slope of the tangent is $\frac{1}{\ell n(\frac{\pi}{2})}$.

At $x = \frac{\pi}{2}$, $y = 1$, therefore the equation of tangent is $\frac{y-1}{x-\frac{\pi}{2}} = -\ell n(\frac{\pi}{2})$ and the equation of normal is $\frac{y-1}{x-\frac{\pi}{2}} = \frac{1}{\ell n(\frac{\pi}{2})}$

b) This limit of the indeterminate form $(\infty - \infty)$ and we have to rewrite by taking the way of common denominator such that

$$\lim_{x\to 0} [\frac{1}{\sin x} - \frac{1}{x}] = \lim_{x\to 0} [\frac{x - \sin x}{x \sin x}] = \lim_{x\to 0} [\frac{1-\cos x}{\sin x + x\cos x}]$$

$$= \lim_{x\to 0} [\frac{\sin x}{2\cos x - x\sin x}] = 0$$

- $\lim_{x\to 0^+} \tan x \, \ell n x = 0 \bullet (-\infty)$

Rewrite the above limit such that $\lim_{x\to 0^+} \tan x \, \ell n x = \lim_{x\to 0^+} \frac{\ell n x}{\cot x} = \frac{-\infty}{\infty}$

Now L'Hospital's rule can be used so that $\lim_{x\to 0^+} \frac{\ell n x}{\cot x} = \lim_{x\to 0^+} \frac{1/x}{-\csc^2 x}$

Simplify and then apply L'Hospital's rule again, i.e $\lim_{x\to 0^+} \frac{1/x}{-\csc^2 x} =$

$$\lim_{x\to 0^+} \frac{\sin^2 x}{-x} = \frac{0}{0} = \lim_{x\to 0^+} \frac{2\sin x \cos x}{-1} = 0$$

Answer of Question 3

a) Let $g(x) = \ell n(x+1) \Rightarrow g`(x) = \frac{1}{x+1} \Rightarrow g``(x) = -\frac{1}{(x+1)^2} \Rightarrow$

$$g```(x) = \frac{2}{(x+1)^3}$$

At x = 0 $\Rightarrow$ g (0) = 0, g`(0) = 1, g``(0) = -1, g```(0) = 2

Using Taylor expansion

$$\ell n(x+1) = g(0) + g`(0)\,x + \frac{g``(0)}{2!}x^2 + \frac{g```(0)}{3!}x^3 = x - \frac{1}{2}x^2 + \frac{1}{3}x^3$$

Therefore $x^3 \ell n(x+1) = x^4 - \frac{1}{2}x^5 + \frac{1}{3}x^6$

$$f(x) = \cos(5x)\sin^2(2x) = \cos(5x)\,\frac{1}{2}[1+\cos(4x)] = \frac{1}{2}\cos 5x + \frac{1}{4}[\cos x + \cos 9x]$$

$$\Rightarrow f^{(n)} = \frac{5^n}{2}\cos\left(5x + \frac{n\pi}{2}\right) + \frac{1}{4}\left[\cos\left(x + \frac{n\pi}{2}\right) + 9^n\cos\left(9x + \frac{n\pi}{2}\right)\right]$$

$$g(x) = \ell n\left[\sqrt[7]{\frac{12 - x - x^2}{25 - x^2}}\right] = \frac{1}{7}[\ell n(3-x) + \ell n(x+4) - \ell n(5-x) - \ell n(5+x)],$$

therefore $g^{(n)} = \frac{(-1)^{n-1}(n-1)!}{7}\left[\frac{(-1)^n}{3-x} + \frac{1}{x+4} - \frac{(-1)^n}{5-x} - \frac{1}{x+5}\right]$