PROBABILITY & STATISTICS

DR/ TAMER RAGEH

1) Introduction

Probability and statistics are used to model uncertainty from a variety of sources, such as incomplete or simplified models. Yet you can build useful models for aggregate or overall behavior of the system in question. These types of models are now universally used across all areas of science, technology, and business.

Probability is the math tool that enables us to measure the reliability of an inference استدلال .

Random Experiment

A *random experiment* is an action or process that leads to one of several possible outcomes but we do not know which one will happen.

2) Sample Space and Events

Sample Space **A list of exhaustive *[don't leave anything out]* and mutually exclusive outcomes is called a *sample space* and is denoted by S. (all outcomes)**

Events

An individual outcome of a sample space is called a **simple event [*cannot break it down into several other events*]**,

We can classify the events to three types:

- 1- Simple Events
- 2- Certain Events
- 3- Impossible Events

Experiment 1: Tossing a coin once.

Sample space: S = {Head or Tail} or we could write:

S = {0, 1} where 0 represents a tail and 1 represents a head.

Experiment 3: Throwing a die.

Sample space: S = {1, 2, 3, 4, 5, 6}

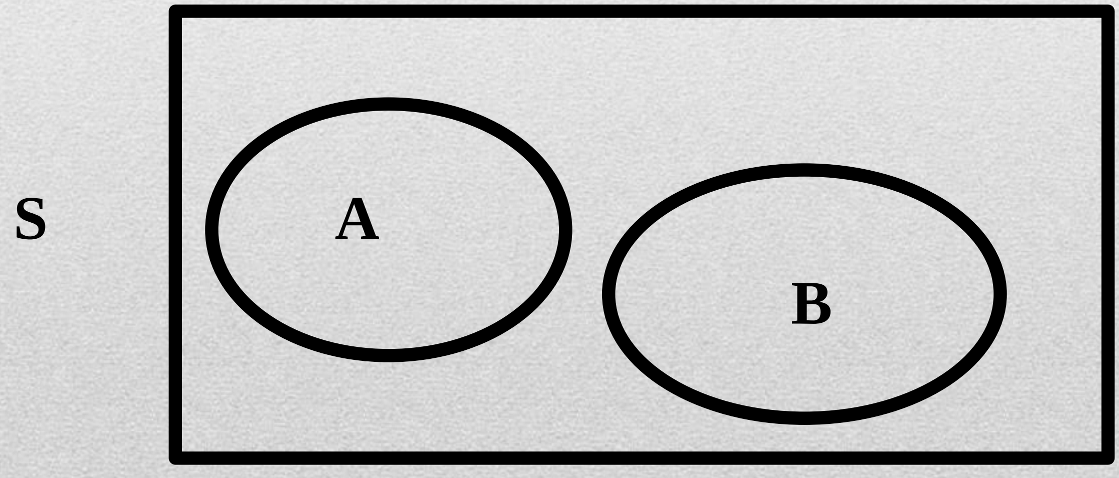
Some events: **Even numbers, E_1 = {2, 4, 6}** **Odd numbers, E_2 = {1, 3, 5}**

The number 1, E_3 = {1} **At least 3, E_4 = {3, 4, 5, 6}**

Venn Diagrams

A universe *S* can be represented geometrically by the set of points inside a rectangle.

In such case subsets of *S* (such as A and B shown shaded in Fig.) are represented by sets of points inside circles. Such diagrams, called *Venn diagrams*, often serve to provide geometric intuition regarding possible relationships between sets.



3) Axioms of Probability

Probability is a number that is assigned to each member of a collection of events from a random experiment that satisfies the following properties:

If S is the sample space and A is any event in a random experiment,

$\boldsymbol{a}) \ \mathbf{0} \leq \boldsymbol{P}(\boldsymbol{A}) \leq \mathbf{1}$

$\boldsymbol{b}) \ \boldsymbol{P}(\boldsymbol{S}) = \mathbf{1}$

$$\boldsymbol{c}) \ \boldsymbol{P}(\boldsymbol{A}) = \frac{\boldsymbol{M}}{\boldsymbol{N}} = \frac{\boldsymbol{Number\ of\ apperance\ of\ A}}{\boldsymbol{Total\ number\ of\ sample\ Space}}$$

Example 1.1: in an experiment of rolling fair dice find the sample space S and the probability of even face.

Solution:

Random Experiment: Rolling a ***die*** Sample Space: S = {1, 2, 3, 4, 5, 6}

Let A the probability of even face A = {2, 4, 6} $P(A) = \frac{3}{6} = \frac{1}{2}$.

Example 1.4 **Rolling 2 die [dice], one red and the other blue and summing 2 numbers on top find the sample space S and the probability of appearance of 2,7 and 10.**

Solution:

Sample Space:

$S = \{2,3, \ldots ,12\}$

Probability Examples:

$$P(2) = \frac{1}{36}$$

$$P(7) = \frac{6}{36}$$

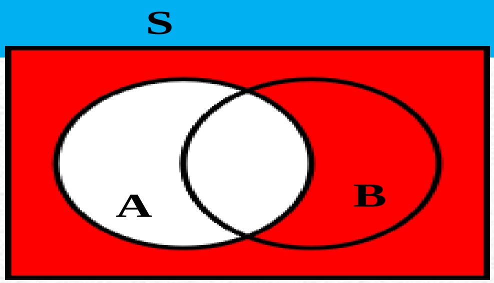
$$P(10) = \frac{3}{36}$$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Complement

A' (read 'A complement') consists of all outcomes not in A (that is, $S - A$).

$$P(A') = 1 - P(A)$$



Independent Events

$P(A \cap B) = P(A) * P(B)$ (*next section (1.4) have more details*)

Example 1.7: A sample space consists of five simple events, E_1, E_2, E_3, E_4, and E_5.

a) If $P(E_1) = P(E_2) = 0.15$, $P(E_3) = 0.4$, $and\ P(E_4) = 2P(E_5)$, find the probabilities of E_4 and E_5.

b) If $P(E_1) = 3P(E_2) = 0.3$, find the probabilities of the remaining simple events if you know that the remaining simple events are equally probable.

a) Since

$$P(S) = P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) = 1$$

$$0.15 + 0.15 + 0.4 + 2P(E_5) + P(E_5) = 1$$

$$P(E_5) = 0.1$$

$$P(E_4) = 0.2$$

b) $P(E_1) = 3P(E_2) = 0.3$ $\qquad$ $P(E_3) = P(E_4) = P(E_5) = x$

$$P(S) = P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) = 1$$

$$0.3 + 0.1 + x + x + x = 0.3 + 3x = 1 \qquad x = 0.2$$

Example 1.8: Hydraulic landing assemblies coming from an aircraft rework facility are each inspected for defects. Historical records indicate that 8% have defects in shafts only, 6% have defects in bushings only, and 2%have defects in both shafts and bushings.

One of the hydraulic assemblies is selected randomly. What is the probability that the assembly has

a) A bushing defect? ***b)*** A shaft or bushing defect?

c) Exactly one of the two types of defects?

d) Neither type of defect?

Solution:

Let B= bushing defect SH=shaft defect

a) P(B)=0.06+0.02=0.08

b) P(B or SH)=0.06+0.08+0.02=0.16

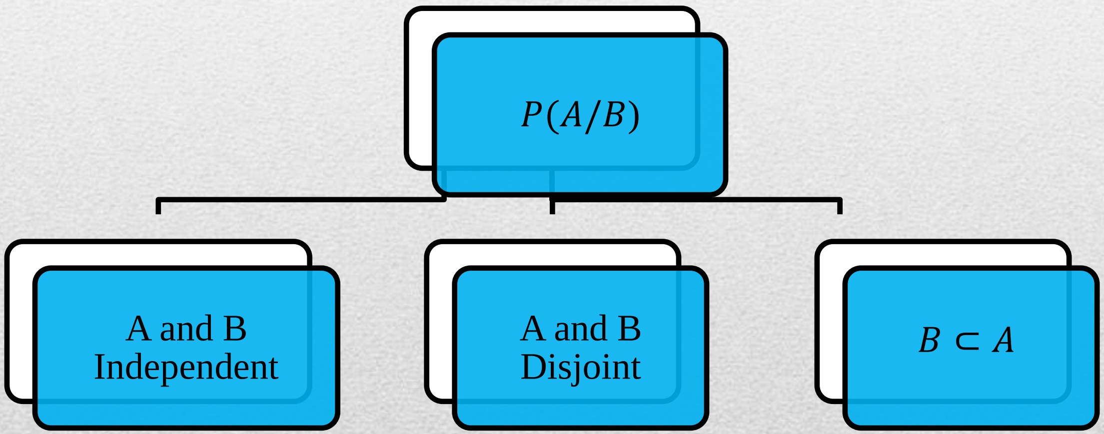
c) P(exactly one defect)=0.06+0.08=0.14

d) P(neither defect)=1-(P(B or SH)=1-0.16=0.84

1.4) Conditional Probability

Conditional probability **is used to determine how two events are related; that is, we can determine the probability of one event *given* the occurrence of another related event.**

$$\boldsymbol{P(A/B) = \frac{P(A \cap B)}{P(B)}} \qquad \boldsymbol{P(B/A) = \frac{P(A \cap B)}{P(A)}}$$



A and B Independent:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(A)P(B)}{P(B)} = P(A)$$

A and B Disjoint:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0}{P(B)} = 0$$

$B \subset A$:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(B)}{P(B)} = 1$$

Example 1.11: A recent survey asked 100 people if they thought women in the armed forces should be permitted to participate in combat. The results are shown in the table

Staff	Yes	No	Total
Males	32	18	50
Females	8	42	50
Total	40	60	100

a) Find the probability that the respondent answered "yes" given that the respondent was a female.

b) Find the probability that the respondent was a male, given that the respondent answered "**no**"

Solution:

Let M = respondent was a male; F = respondent was a female;

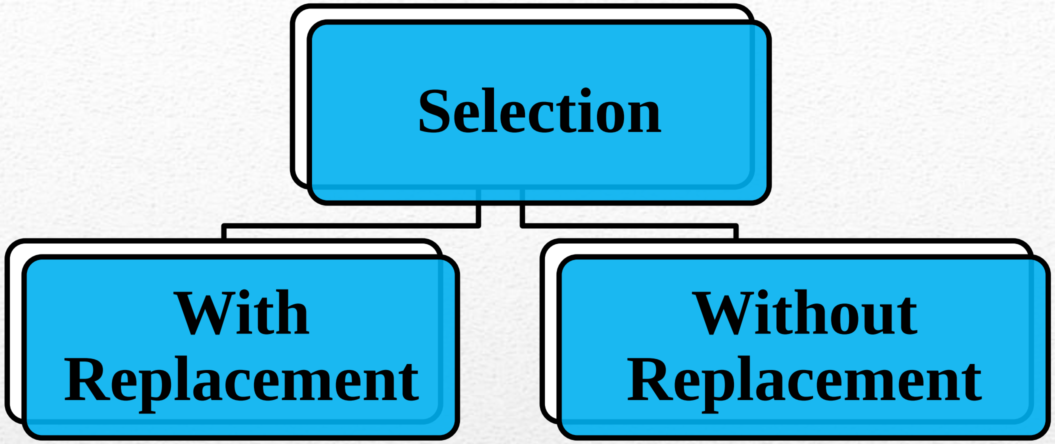
Y = respondent answered "yes"; N = respondent answered "no".

$$a)\ P(Y/F) = \frac{[P(\ F\ and\ Y)]}{P(F)} = \frac{\left[\frac{8}{100}\right]}{\left[\frac{50}{100}\right]} = \frac{4}{25} \quad b)\ P\left(\frac{M}{N}\right) = \frac{[P(N\ and\ M)]}{P(N)} = \frac{\left[\frac{18}{100}\right]}{\left[\frac{60}{100}\right]} = \frac{3}{10}$$

5) Independence

Two events A and B are called independent if the occurrence of B does not change the probability that A occurs, or the occurrence or nonoccurrence of one does not affect the occurrence or nonoccurrence of the other. In this situation events A and B are called *statistically independent* or simply *independent*. For instance, successive selections of a ball with replacement are an example of independent events. Mathematically, we say that two events A and B are independent if

$$P(A \cap B) = P(A) * P(B)$$



With replacement

Sampling is called with replacement when a unit selected at random from the population is returned to the population and then a second element is selected at random

Without replacement

Sampling is called without replacement when a unit is selected at random from the population and it is not returned to the main lot.

Example 1.17: **An urn contains 6 white balls and 4 black balls. Find the probability that 4-successive selected balls are all white**

Solution:

1-With replacement: $P = \frac{6}{10}\frac{6}{10}\frac{6}{10}\frac{6}{10} = \frac{81}{625}$

2-Without replacement $P = \frac{6}{10}\frac{5}{9}\frac{4}{8}\frac{3}{7} = \frac{1}{14}$

Example 1.24: An urn contains 7 red marbles and 3 white marbles. Three marbles are drawn from the urn **one after the other**

i) Find the probability p that all three are red

ii) Find the probability p that all three are white

iii) Find the probability p that the first two are red and the third is white.

Solution:

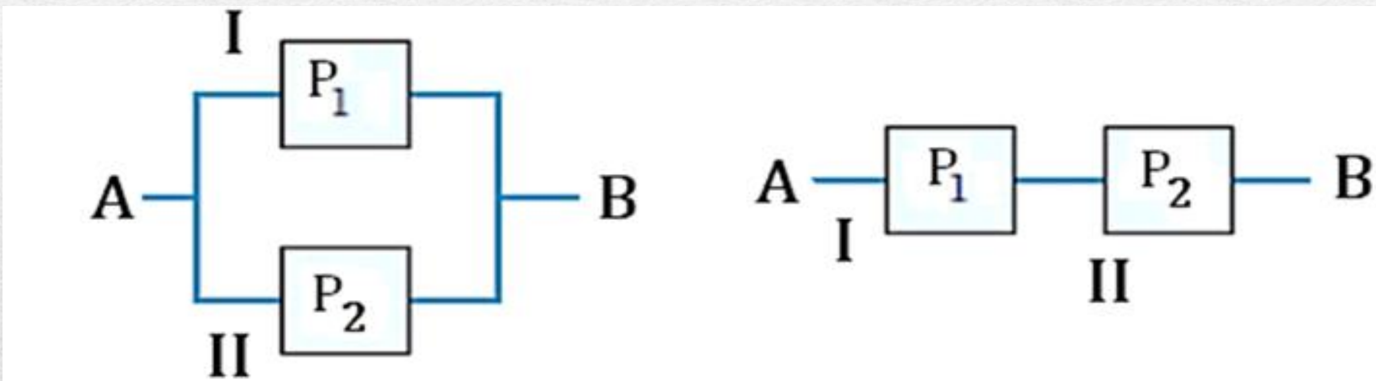
$$i)\ \ p = \frac{7}{10}\frac{6}{9}\frac{5}{8} = \frac{7}{24}$$

$$ii)\ \ p = \frac{3}{10}\frac{2}{9}\frac{1}{8} = \frac{1}{120}$$

$$iii)\ \ p = \frac{7}{10}\frac{6}{9}\frac{3}{8} = \frac{7}{40}$$

Electrical Circuit (Switches)

Parallel | **Series**



Union (or)

$$P(I \cup II) = P(I) + P(II) - P(I \cap II)$$

$$= P(I) + P(II) - P(I).P(II)$$

Intersection (and)

$$P_{AB} = P(I \cap II) = P(I).P(II)$$

Another method for parallel

$$P(I \cup II) = 1 - P(I' \cap II') = 1 - p(I').P(II')$$

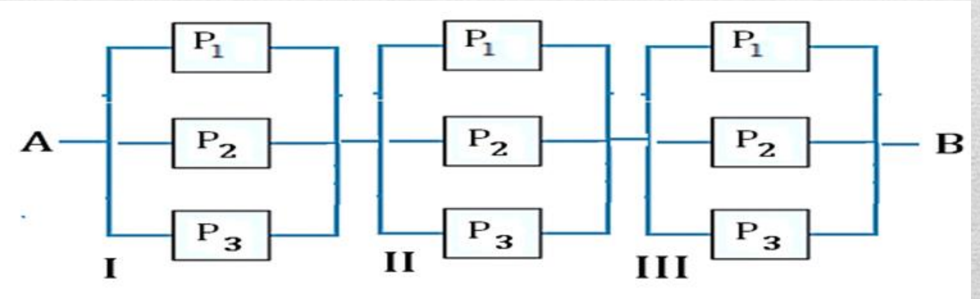
$$I\ , II \rightarrow independent \rightarrow I', II' indepen.$$

$$\therefore P(I \cup II) = 1 - ([1 - P(I)] * [1 - P(II)])$$

If we have more than 2 devices

$$P(I \cup II \cup III \cup \cdots) = 1 - ([1 - P(I)] * [1 - P(II)] * [1 - P(III)] \ldots \ldots..)$$

Example 1.25: What is the probability that the circuit operates?



$$P(I) = 1 - [(1 - P_1) * (1 - P_2) * (1 - P_3)] = P(II) = P(III)$$
$$P_{AB} = P(I).P(II).P(III) = [(1 - (1 - P_1)(1 - P_2)(1 - P_3)]^3$$

$$\boldsymbol{IF\ the\ prob\ of\ flow\ is\ required\ to\ be\ at\ least\ \frac{1}{8}}$$

and all compenent are indentical with probability p

compute the lower bound of p

$$p_1 = p_2 = p_3 = p$$

$$P_{AB} = (1-(1 - p)^3)^3 \geq \frac{1}{8}$$

$$\therefore (1 -(1 - p)^3) \geq \frac{1}{2} \qquad -(1 - p)^3 \geq -\frac{1}{2} \qquad (1 - p)^3 \leq \frac{1}{2}$$

$$(1 - p) \leq \sqrt[3]{\frac{1}{2}} \quad \rightarrow \quad -p \leq \sqrt[3]{\frac{1}{2}} - 1 \quad \rightarrow \quad \therefore p \geq (1 - \sqrt[3]{\frac{1}{2}})$$

$$p \geq 0.206$$
$$P_{AB} = (1-(1 - 0.206)^3)^3$$

THANK YOU