PROBABILITY
&
STATISTICS

Total probability & Bayes' Low

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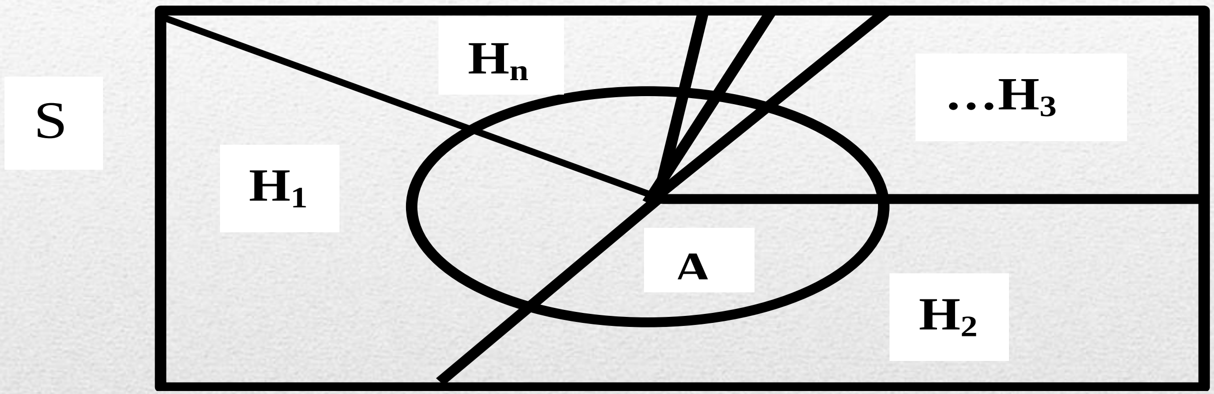
6) Total probability

If the events H_1, H_2, . . . , H_k constitute a partition of the sample space S such that $P(H_i) \neq 0$ for $i = 1, 2, \ldots, k$, then for any event A of S,
If we have an event A that happen only from the union of mutually exclusive events (disjoint) the P(A) is called total probability



Conditions must achieve

$$\sum_{i=1}^{n} P(H_i) = 1$$

total partition

$$P(H_i \cap H_{i+1}) = 0$$

no intersection

(mutually exclusive)

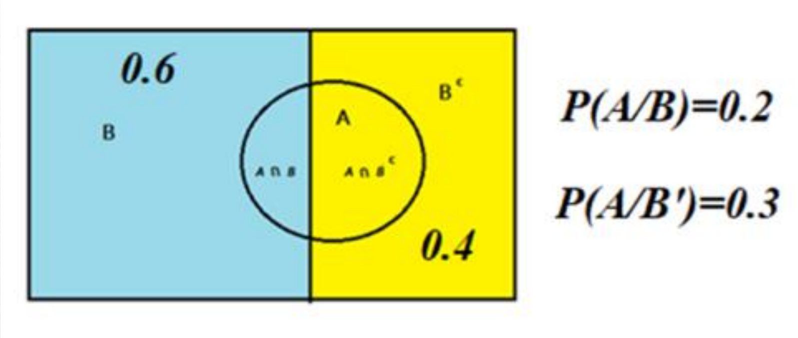
$$\boldsymbol{P(A) = P(A \cap H_1) + P(A \cap H_1) + P(A \cap H_1) + \cdots}$$

$$\because \quad \boldsymbol{P(A / H_1) = \frac{P(A \cap H_1)}{P(H_1)}}$$

$$\boldsymbol{P(A) = P(A/H_1) * P(H_1) + P(A/H_2) * P(H_2) + P(A/H_3) * P(H_3) + \cdots}$$

$$\boldsymbol{\therefore P(A) = \sum_{i=1}^{n} P(A/H_i) * P(H_i)}$$

Example 1.26: the Veen diagram shown that B and $B^c = B'$ constitute a partition of the sample space A, find P(A) (*total probability*).



Solution:

$$P(A) = P(A \cap B) + P(A \cap B')$$

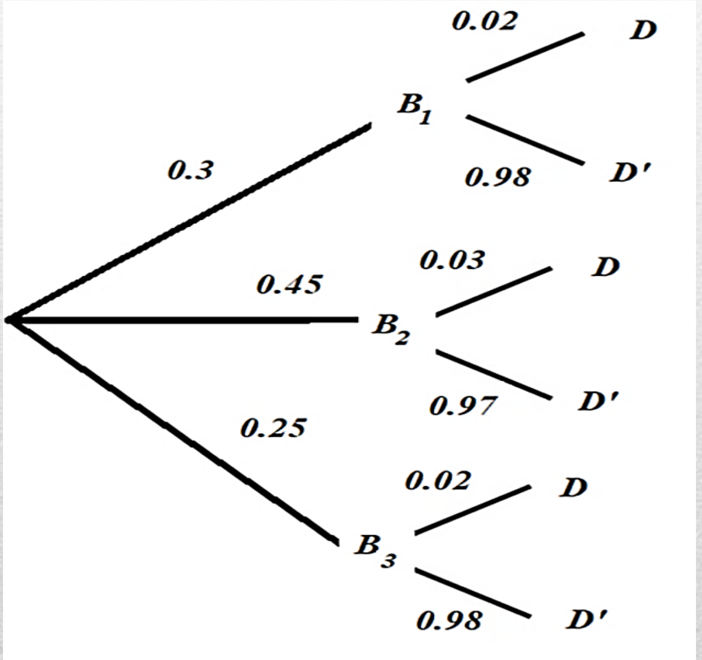
$$\because \quad P(A / B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A) = P(A/B) * P(B) + P(A/B') * P(B')$$

$$= 0.2 * 0.6 + 0.3 * 0.4 = 0.24$$

Example 1.27: **In a certain assembly plant, three machines, B_1, B_2, and B_3, make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected. What is the probability that it is defective?**

Solution:



Consider the following events:

- D: the product is defective,
- B_1: the product is made by machine B_1,
- B_2: the product is made by machine B_2,
- B_3: the product is made by machine B_3.

$P(D) = P(B_1)P(D/B_1) + P(B_2)P(D/B_2) + P(B_3)P(D/B_3)$.

Referring to the tree diagram, we find that the three branches give the probabilities

$P(B_1)P(D/B_1) = (0.3)(0.02) = 0.006,$

$P(B_2)P(D/B_2) = (0.45)(0.03) = 0.0135,$

$P(B_3)P(D/B_3) = (0.25)(0.02) = 0.005,$

and hence

$$P(D) = 0.006 + 0.0135 + 0.005 = 0.0245.$$

7) Bayes' Theorem

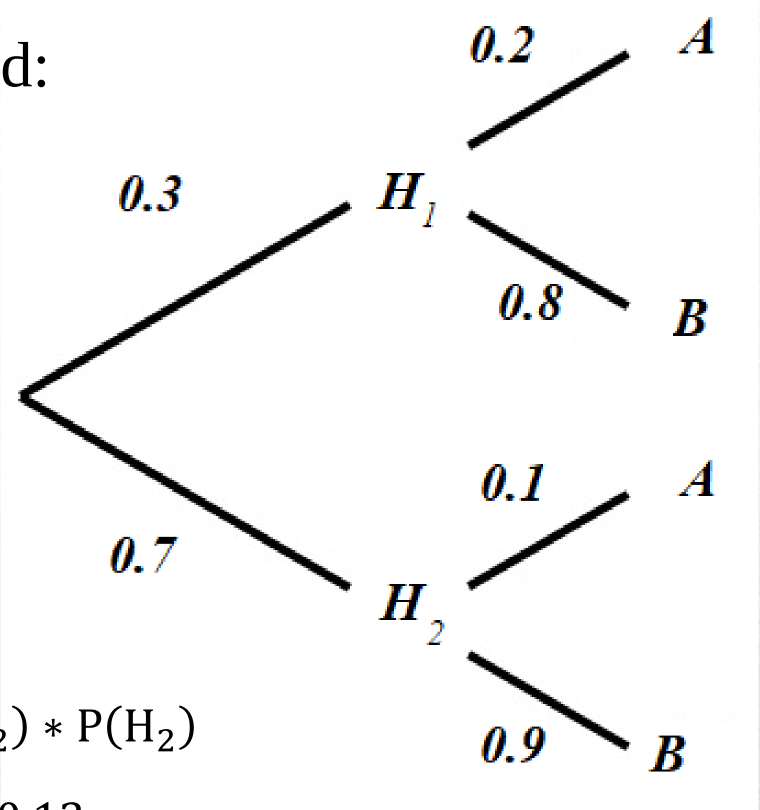
$$P(H_1 / A) = \frac{P(A \cap H_1)}{P(A)}$$

$$\because \quad P(A/H_1) = \frac{P(A \cap H_1)}{P(H_1)} \qquad \because \quad P(A) = \sum_{i=1}^{n} P(A/H_i) * P(H_i)$$

$$\therefore P(H_1 / A) = \frac{P(A/H_1) * P(H_1)}{\sum_{i=1}^{n} P(A/H_i) * P(H_i)}$$

Example 1.28: Using the tree diagram find:

$P(H_1)$, $P(H_2)$, $P(A/H_1)$,
$P(A/H_2)$, $P(B/H_1)$, $P(B/H_2)$,
$P(A)$, $P(B)$, $P(H_1/A)$,
$P(H_2/A)$, $P(H_1/B)$, $PH_2/B)$.



Solution:

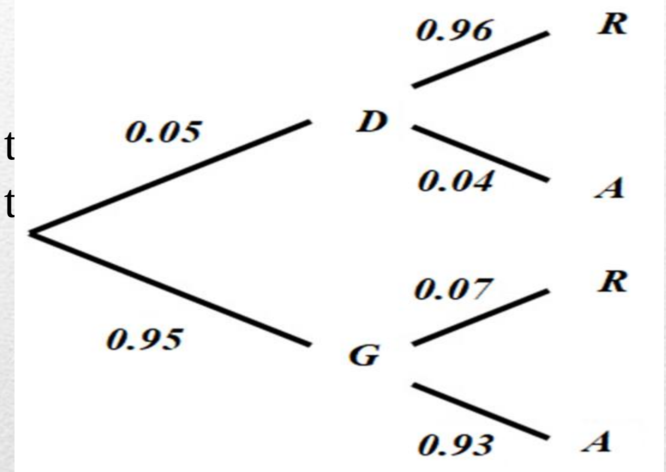
$P(H_1)=0.3$ $P(H_2)=0.7$ $P(A/H_1)=0.2$
$P(A/H_2)=0.1$ $P(B/H_1)=0.8$ $P(B/H_2)=0.9$

$$P(A) = P(A/H_1) * P(H_1) + P(A/H_2) * P(H_2)$$
$$= (0.2 * 0.3) + (0.1 * 0.7) = 0.13$$
$$P(B) = P(B/H_1) * P(H_1) + P(B/H_2) * P(H_2)$$
$$= (0.8 * 0.3) + (0.9 * 0.7) = 0.87$$
$$P(H_1/A) = \frac{P(A/H_1) * P(H_1)}{P(A)} = \frac{0.2 * 0.3}{0.13} = 0.46$$

Similarly $P(H_2/A)$, $P(H_1/B)$, $P(H_2/B)$.

Exa [illegible] A company produces machine components which pass through an automatic
testing machine. 5% of the components entering the testing machine are defective. However, the machine is not entirely reliable. If a component is defective there is 4% probability that it will not be rejected. If a component is not defective there is 7% probability that it will be rejected.

- What is the probability that all the components are rejected?
- What is the probability that the components defective given t
- What is the probability that the components defective given t



Solution:

Let
D represent a defective component
G a good component.
R represent a rejected component
A an accepted component.

a) can be answered directly using a tree diagram.

$$P(R) = P(R/D) * P(D) + P(R/G) * P(G) = 0.96 * 0.05 + 0.07 * 0.95 = 0.1145$$

$$b)\ P(D/R) = \frac{P(D \cap R)}{P(R)} = \frac{P(R/D) * P(D)}{P(R/D) * P(D) + P(R/G) * P(G)} = \frac{0.96 * 0.05}{0.1145} = 0.419$$

$$c)\ P(D/A) = \frac{P(D \cap A)}{P(A)} = \frac{P(A/D) * P(D)}{1 - P(R)} = \frac{0.04 * 0.05}{1 - 0.1145} = 0.0022586$$

H.W

Assignment (1)

THANK YOU