PROBABILITY & STATISTICS

Lec: Random Variables

Dr/ Tamer Rageh

Random Variables

Discrete Random Variables

(Mean, Variance and Standard deviation)

- The Binomial Distribution
- Poisson Distributions
- Geometric Distribution

Continuous Random Variables

(Mean, Variance and Standard deviation)

- Uniform Distribution
- Exponential Distribution
- Normal Distribution

Random Variables

A *random variable* is a function or rule that assigns a number to each outcome of an experiment. Basically it is just a symbol that represents the outcome of an experiment.

Two Types of Random Variables

- Discrete Random Variables
- Continuous Random Variables

Discrete Random Variables

A **discrete random variable** X can take a countable number of distinct values like 1, 2, 3, 4,… .Usually counts.

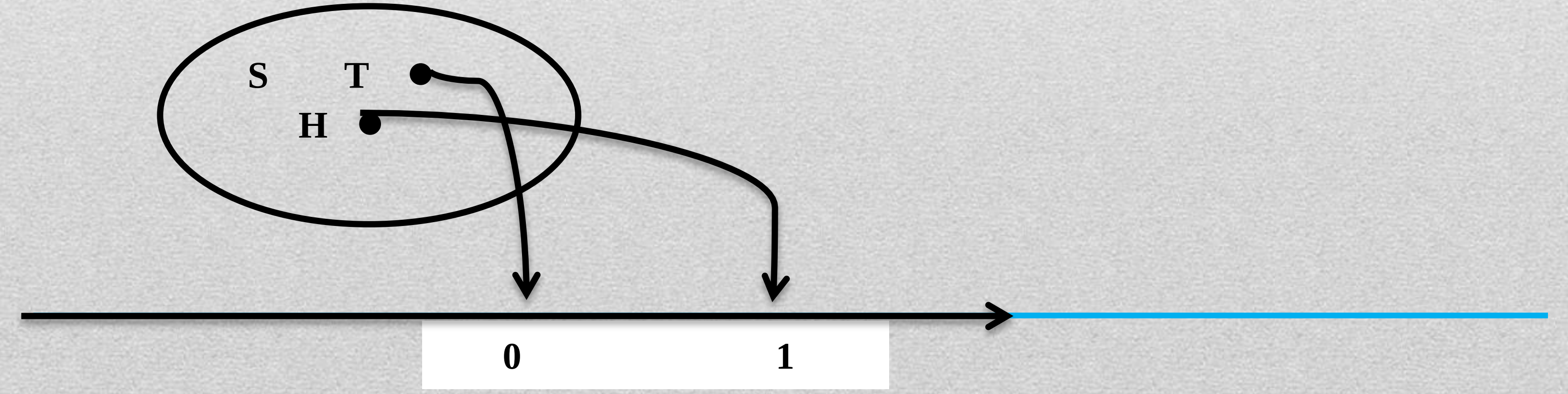
Definition (Discrete Random Variable)**.** We say a random variable is a **discrete random variable** if its space is either finite or countable.

Experiment 1: Tossing a coin once.

S = {Head or Tail}, if the random variable is number of heads.

Outcomes	Random Variable
H	1
T	0

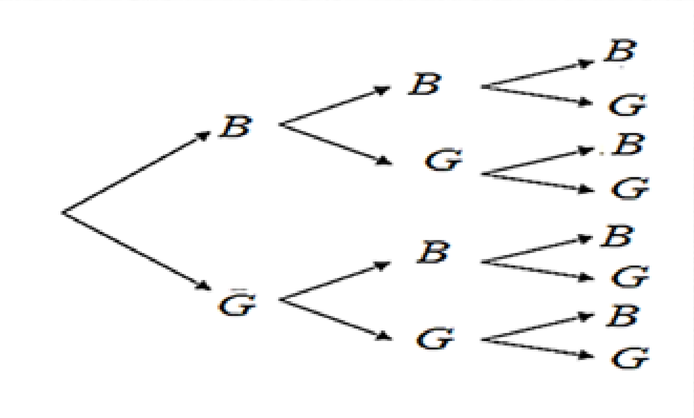
The Random Variable $X = x, \quad x = 0,1$



Example 2.1: A family consisted of 3 children assume the random variable X gives the number of boys. Find the values of random variable and the corresponding probability density function.

Solution:

Sample Space: S = {BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG}. Each outcome is equally likely, if the random variable is number of boys.



The Random Variable $X = x, \quad x = 0,1,2,3.$

The event that the family has 3 boys $A_3 = \{BBB\}$,
The event that the family has 2 boys $A_2 = \{BBG, BGB, GBB\}$,
The event that the family has 1 boy $A_1 = \{BGG, GBG, GGB\}$,
The event that the family has no boys $A_0 = \{GGG\}$.

Outcomes	Random Variable
BBB	3
BBG	2
BGB	
GBB	
BGG	1
GBG	
GGB	
GGG	0

$$P(X = 0) = \frac{1}{8}$$

$$P(X = 1) = \frac{3}{8}$$

$$P(X = 2) = \frac{3}{8}$$

$$P(X = 3) = \frac{1}{8}$$

The Cumulative Distribution Function

The **cumulative distribution function** $F(x)$ of a discrete random variable X with probability distribution $f(x)$ is

$$F(x) = P(X \le x) = \sum_{X \le x} f(X)$$

Example 2.4: A discrete random variable X has the following probability distribution:

X	1	2	3	4
P(X)	0.4	0.3	0.2	C

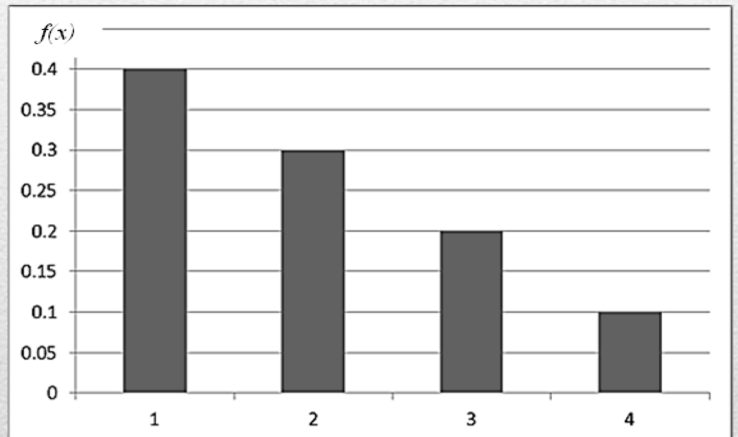
Find the value of C, find $F(x_i)$ and graph both f(x) and F(x)

Solution:

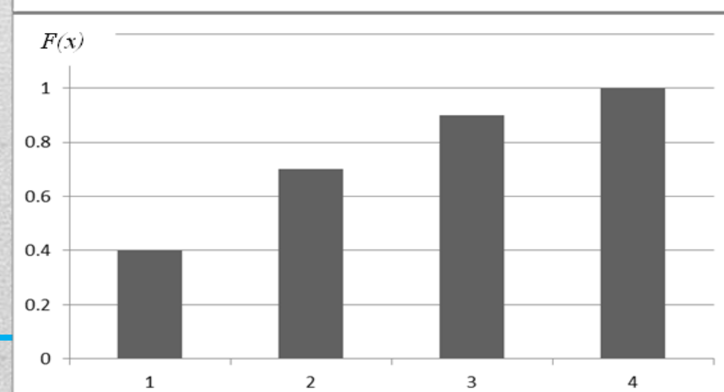
$$\sum_{i=1}^{n} P(X_i) = 1$$

$$0.4 + 0.3 + 0.2 + C = 1$$

$$C = 0.1$$



$F(1) = 0.4$,
$F(2) = 0.7$,
$F(3) = 0.8$,
$and\ F(4) = 1.$



Mean and Variance of A discrete Random Variable

Mean (Expected Value)

The mean, ***expected value***, or ***expectation*** of a random variable X is written as $E(X)$ or μ_X. The expected value of a discrete random variable is a measure of central location. The expected value has the formula:

$$E(X) = \mu = \sum_{i=1}^{n} X_i \, P(X_i).$$

Example 2.6:

Let the random variable X of the discrete type have the pdf given by the table:

X	1	2	3	4
P(X)	0.4	0.1	0.3	0.2

$$E(X) = \sum_{i=0}^{n} x_i \, P(x_i)$$

$$= \left(1 * \frac{4}{10}\right) + \left(2 * \frac{1}{10}\right) + \left(3 * \frac{3}{10}\right) + \left(4 * \frac{2}{10}\right) = 2.3.$$

Example 2.7: A player tosses a fair die. If a prime number occurs he wins that number of dollars, but if a non-prime number occurs he loses that number of dollars.is this game fair to the player.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, …

O/P	R.V	P(x)
1	-1	1/6
2	2	1/6
3	3	1/6
4	-4	1/6
5	5	1/6
6	-6	1/6

Solution:

The possible outcomes of the game with their respective probabilities are as follows:

x	-6	-4	-1	2	3	5
P(x)	1/6	1/6	1/6	1/6	1/6	1/6

$$E(X) = \sum_{i=0}^{n} x_i\, P(x_i) = \left(-6 * \frac{1}{6}\right) + \left(-4 * \frac{1}{6}\right) + \left(-1 * \frac{1}{6}\right)$$

$$+ \left(2 * \frac{1}{6}\right) + \left(3 * \frac{1}{6}\right) + \left(5 * \frac{1}{6}\right) = -\frac{1}{6}$$

Thus the game is unfavorable to the player since the expected value is negative.

Variance:

The variance and associated standard deviation are used to measure the variability of the random variable. The formula for the variance is

$$\boldsymbol{Var(X) = \sigma^2 = \sum_{i=1}^{n} (X_i - E(X))^2 P(X_i)}.$$

If X is a random variable with mean μ, then the variance of X, denoted by Var(X), is defined by:

$$Var(X) = E[(X-\mu)^2]$$

An alternative formula for $Var(X)$ can be derived as follows:

$$Var(X) = E[(X-\mu)^2]$$

$$= E[X^2 - 2\mu X + \mu^2]$$

$$= E[X^2] - E[2\mu X] + E[\mu^2]$$

$$= E[X^2] - 2\mu E[X] + \mu^2$$

$$= E[X^2] - 2\mu^2 + \mu^2$$

$$\boldsymbol{= E\left[X^2\right] - \mu^2}$$

➢ Standard Deviation:

Standard deviation is a measure that is used to quantify the amount of variation or dispersion of a set of data values. A low standard deviation indicates that the data points tend to be close to the mean of the set, while a high standard deviation indicates that the data points are spread out over a wider range of values.

Standard Deviation Formula:

$$\sigma = \sqrt{\sigma^2}.$$

Example 2.8: A random number generator produces sequences of independent digits, each of which is as likely to be any digit from 0 to 9 as any other. If X denotes any single digit, find E(X),Var(X) and σ.

Solution:

X	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
P(X=*x*)	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**

$$E(X) = \sum_{i=0}^{n} x_i \, P(x_i)$$

$$= \big((0*0.1)+(1*0.1)+(2*0.1)+(3*0.1)+(4*0.1)+(5*0.1) + (6*0.1)+(7*0.1)+(8*0.1)+(9*0.1)\big) = 4.5$$

$$Var(X) = E[X^2]-(E[X])^2 = 28.5-(4.5)^2 = 8.25.$$

X_i	**$P(X_i)$**	**$X_iP(X_i)$**	**$X^2_iP(X_i)$**
0	**0.1**	**0**	**0**
1	**0.1**	**0.1**	**0.1**
2	**0.1**	**0.2**	**0.4**
3	**0.1**	**0.3**	**0.9**
4	**0.1**	**0.4**	**1.6**
5	**0.1**	**0.5**	**2.5**
6	**0.1**	**0.6**	**3.6**
7	**0.1**	**0.7**	**4.9**
8	**0.1**	**0.8**	**6.4**
9	**0.1**	**0.9**	**8.1**
		E(X)=4.5	**$E(X^2)$=28.5**

$$\sigma = \sqrt{\sigma^2} = \sqrt{8.25} = 2.87.$$

Example 2.9: A discrete random variable X has the following probability distribution:

X	1	2	3	4	5	6	7
P(X)	C	$2C$	$2C$	$3C$	C^2	$2C^2$	$7C^2 + C$

Find the value of C. Also find the mean of distribution.

Solution:

$\sum_{i=1}^{n} P(X_i) = 1 = c + 2c + 2c + 3c + c^2 + 2c^2 + 7c^2 + c$

$$10c^2 + 9c - 1 = (10c - 1)(c + 1) = 0$$

$$c = 0.1 \qquad\qquad c = -1 \quad rejected$$

$$E(X) = \mu = \sum_{i=1}^{n} X_i\, P(X_i)$$

$$= (1 * 0.1) + (2 * 0.2) + (3 * 0.2) + (4 * 0.3) + (5 * 0.01) + (6 * 0.02) + (7 * 0.17) = 3.66\,.$$

THANK YOU