PROBABILITY
&
STATISTICS

Lec: Binomial &Poisson Distribution

Dr/ Tamer Rageh

The Binomial Distribution

The requirements for using the binomial distribution are as follows:

- The outcome is determined completely by **chance**.
- There are only **two** possible outcomes.
- All trials have the **same probability** for a particular outcome in a single trial. That is, the probability in a subsequent trial is independent of the outcome of a previous trial. Let this constant probability for a single trial be p.
- The number of trials, **n**, must be **fixed**, regardless of the outcome of each trial.

IF the random variable X has Binomial distribution, then we write P(X)= b(X;n,p):

$$\boldsymbol{P(X = x) = \binom{n}{x} p^x q^{(n-x)}}$$

$$\boldsymbol{x = 0, 1, 2, \ldots, n}$$

n – Number of trials;

x – Number of successes;

p – Probability of a success in one trial

q – probability of failures q = 1 - P.

Consider the following scenarios:

- The number of heads/tails in a sequence of coin flips.
- Vote counts for two different candidates in an election.
- The number of male/female employees in a company.
- The number of successful sales calls.
- The number of defective products in a production run.
- The number of days in a month your company's computer network experiences a problem.

1) Mean $\mu = E(X) = np$
2) Variance $\sigma^2 = npq$
3) Standard Deviation $\sigma = \sqrt{npq}$

Example 2.11: Team A has probability 2/3 of winning whenever it plays. If A plays 4 games, find the probability that A wins

(i) Exactly 2 games (ii) At least 1 game (ii) More than half of the games.

Solution:

Here $n = 4, p = 2/3$ and $q = 1 - p = 1/3$

(i) Exactly 2 games,

$$P(2\ wins) = P(X = 2) = b\left(2; 4, \frac{2}{3}\right) = \binom{4}{2}\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2 = 6 * \left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2 = \frac{8}{27}$$

(ii) At least 1 game,

$$P(X \geq 1) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$$

$$= 1 - P(X = 0) = 1 - \binom{4}{0}\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^4 = 1 - \frac{1}{80} = \frac{80}{81}$$

(iii) More than half of the games.

$$P(X > 2) = P(X = 3) + P(X = 4)$$

$$= \binom{4}{3}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^1 + \binom{4}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^0 = 80/243$$

Example 2.12: Eggs are packed in boxes of 12. The probability that each egg is broken is 0.35 Find the probability that there are less than 3 broken eggs

Solution:

Here $n = 12, p = 0.35$ and $q = 1 - p = 0.65$

$$P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= \binom{12}{0} 0.35^0 0.65^{12} + \binom{12}{1} 0.35^1 0.65^{11} + \binom{12}{2} 0.35^2 0.65^{10}$$

$$= 0.0151$$

2.2.5) The Poisson distribution

This is a discrete distribution that is used in two situations. It is used, when certain conditions are met, as a probability distribution in its own right, and it is also used as a convenient approximation to the binomial distribution in some circumstances. The distribution is named for **S.D. Poisson**, a French mathematician of the nineteenth century.

Consider the following scenarios:

- The hourly number of customers arriving at a bank
- The daily number of accidents on a particular stretch of highway
- The hourly number of accesses to a particular web server
- The daily number of emergency calls in Dallas
- The number of typos in a book
- The monthly number of employees who had an absence in a large company
- Monthly demands for a particular product

Examples of occurrences to which the Poisson distribution often applies include counts from a *Geiger counter*, *collisions of cars* at a specific intersection under specific conditions, *flaws in a casting,* and *telephone calls* to a particular telephone or office under particular conditions. For the Poisson distribution to apply to these outcomes, they must occur randomly.

$$P(X) = \frac{\lambda^x e^{-\lambda}}{x!} \quad x = 0, 1, 2, \ldots$$

where λ is the average number of occurrences per base unit, and t is the number of base units inspected

1) Mean $\mu = E(X) = \lambda$

2) Variance $\sigma^2 = \lambda$

3) Standard Deviation $\sigma = \sqrt{\lambda}$

Example 2.15: new cases of $\underline{H_5\ N_1}$ Virus in New England are occurring at a rate of about 2 per month, Find the probabilities that: 0,1, 2, 3, 4, 5, 6 cases will occur in New England in the next month

$$P(x=0)=\frac{2^0e^{-2}}{0!}=0.135$$

$$P(x=1)=\frac{2^1e^{-2}}{1!}=0.27$$

$$P(x=2)=\frac{2^2e^{-2}}{2!}=0.27$$

$$P(x=3)=\frac{2^3e^{-2}}{3!}=0.18$$

$$P(x=4)=\frac{2^4e^{-2}}{4!}=0.09$$

Example 2.18: The number of meteors(الشهب) found by a radar system in any 30-second interval under specified conditions averages 1.81. Assume the meteors appear randomly and independently.

a) What is the probability that no meteors are found in a one-minute interval?

b) What is the probability of observing at least five but not more than eight meteors in two minutes of observation?

Solution:

$a)$

λ	$time$
1.81	$30\ second$
$\lambda_{new\ 1}$	$60\ second$

$$\lambda_{new\ 1} = 3.62/min$$

$$P(X = x) = \frac{3.62^x e^{-\lambda}}{x!} \qquad P(0) = \frac{3.62^0 e^{-3.62}}{0!} = 0.0268$$

$b)$

λ	$time$
1.81	$30\ second$
$\lambda_{new\ 2}$	$120\ second$

$$\lambda_{new\ 2} = 7.24/min$$

$$P(5 \leq X \leq 8) = P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8) = 0.545$$

Example 2.16: Message arrives at a computer at an average rate of 15 messages / second. The number of messages that arrive in 1 second is known to be a Poisson random variable.

a) Find the probability that no messages arrive in 1 second

b) Find the probability that more than 10 messages arrive in a 1- second period

Solution:

$$a)\ P(0) = \frac{\lambda^0 e^{-\lambda}}{0!} = e^{-15} = 3.06 * 10^{-7}$$

$$b)\ P(x > 10) = 1 - P(n \leq 9) = 1 - \sum_{i=0}^{9} \frac{15^i e^{-15}}{i!} = 0.8815$$

THANK YOU