

مسائل محلولة

$$\vec{PQ} = (1, 4)$$

$$\vec{PQ} = \vec{Q} - \vec{P}$$

$$\vec{P} = \vec{Q} - \vec{PQ}$$

$$\vec{Q} = (1, 1) \text{ اذا كانت } \textcircled{A}$$

$$\vec{P} = (1, 1) - (1, 4) = (0, -3)$$

$$\vec{Q} = (3, -2) \text{ اذا كانت } \textcircled{B}$$

$$\vec{P} = (3, -2) - (1, 4) = (2, -6)$$

$$\vec{Q} = (-2, 3) \text{ اذا كانت } \textcircled{C}$$

$$\vec{P} = (-2, 3) - (1, 4) = (-3, -1)$$

$$\vec{Q} = (0, -3) \text{ اذا كانت } \textcircled{D}$$

$$\vec{P} = (0, -3) - (1, 4) = (-1, -7)$$

$$\vec{AB} = (3, 2) - (0, 0) = (3, 2) \textcircled{A}$$

$$\vec{CD} = (2, 0) - (-1, -2) = (3, 2)$$

$$|\vec{AB}| = \sqrt{9+4} = \sqrt{13}$$

$$|\vec{CD}| = \sqrt{9+4} = \sqrt{13}$$

$$\tan \theta_1 (\text{for } \vec{AB}) = \frac{2}{3}$$

$$\tan \theta_2 (\text{for } \vec{CD}) = \frac{2}{3}$$

$$\therefore |\vec{AB}| = |\vec{CD}|$$

$$\therefore \theta_1 = \theta_2$$

$$\therefore \vec{AB} \sim \vec{CD}$$

مثال: D, C, B, A :-

$$\vec{AB} = \vec{B} - \vec{A} = (2, 0) - (3, 4) \textcircled{A}$$

$$= (-1, -4)$$

$$\vec{AB} = (-1, 2) - (1, 1) = (-2, 1) \textcircled{B}$$

$$\vec{AB} = (1, -2) - (-1, -2) \textcircled{C}$$

$$= (2, 0)$$

$$\vec{AB} = (2, 1) - (2, 1) \textcircled{D}$$

$$= (0, 0)$$

$$\vec{PQ} = (-2, 3) \textcircled{A}$$

$$\vec{PQ} = \vec{Q} - \vec{P}$$

$$\vec{Q} = \vec{PQ} + \vec{P}$$

$$\vec{P} = (0, 0) \text{ اذا كانت } \textcircled{A}$$

$$\vec{Q} = (-2, 3) + (0, 0) = (-2, 3)$$

$$\vec{P} = (-3, 2) \text{ اذا كانت } \textcircled{B}$$

$$\vec{Q} = (-2, 3) + (-3, 2) = (-5, 5)$$

$$\vec{P} = (3, -2) \text{ اذا كانت } \textcircled{C}$$

$$\vec{Q} = (-2, 3) + (3, -2) = (1, 1)$$

$$\vec{P} = (-2, -5) \text{ اذا كانت } \textcircled{D}$$

$$\vec{Q} = (-2, 3) - (-2, -5)$$

$$= (0, 8)$$

$\vec{AB} = (5, 4) - (2, 3) = (3, 1)$
 $\vec{CD} = (4, 2) - (7, -2) = (-3, 4)$
 $\vec{AB} + \vec{CD} = (3, 1) + (-3, 4) = (0, 5)$

$\vec{AB} - \vec{CD} = (3, 1) - (-3, 4) = (6, -3)$

$\vec{AC} = (7, -2) - (2, 3) = (5, -5)$

$\vec{DB} = (5, 4) - (4, 2) = (1, 2)$

$\vec{AC} + \vec{DB} = (5, -5) + (1, 2) = (6, -3)$

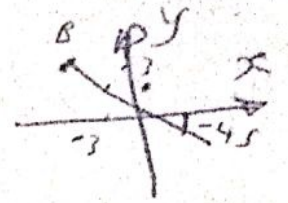
$\vec{AB} + \vec{BC} + \vec{CD} + \vec{DA}$
 $= (3, 1) + (2, -6) + (-3, 4) + (-2, 1)$
 $= (3 + 2 - 3 - 2, 1 - 6 + 4 + 1)$
 $= (0, 0)$

$3\vec{A} = 3(1, 4) = (3, 12)$

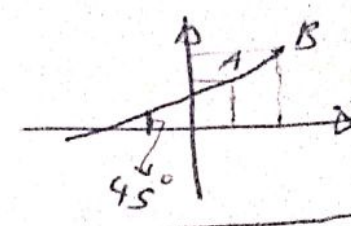
$-2\vec{BA} = -2[(1, 4) - (3, -1)]$
 $= -2(-2, 5) = (4, -10)$

$\vec{A} - \vec{B} = (1, 4) - (3, -1) = (-2, 5)$

$\vec{AB} = (-3, 3) - (-2, 2) = (-1, 1)$
 $\tan \alpha = \frac{1}{-1} = -1$
 $\therefore \alpha = -45^\circ$



$\vec{AB} = (3, 3) - (2, 2) = (1, 1)$
 $\tan \alpha = \frac{1}{1} = 1$
 $\alpha = 45^\circ$



$\vec{AB} = (3, -1) - (0, 3) = (3, -4)$

~~$\vec{CD} = (5, -4) - (1, 0) = (4, -4)$~~

$\vec{CD} = (-9, -4) - (-6, 0) = (-3, -4)$

$|\vec{AB}| = \sqrt{9 + 16} = 5$

$\tan \theta_1 (\text{for } \vec{AB}) = \frac{-4}{3}$

$|\vec{CD}| = \sqrt{9 + 16} = 5$

$\tan \theta_2 (\text{for } \vec{CD}) = \frac{-4}{-3}$

بالرغم من ان قيمتي الجيبين متساوية
 ولكن الزوايا مختلفة فانه
 الجيبين غير متكافئين

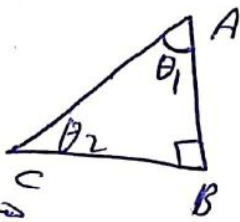


نقطتين في الفضاء

$$\vec{p} = \vec{BC} \perp \vec{AB}$$

$$\vec{AB} \perp \vec{BC}$$

$$\vec{AC} = (-1, -3)$$



$$\begin{aligned} \cos \theta_1 &= \frac{|\vec{AB} \cdot \vec{AC}|}{|\vec{AB}| |\vec{AC}|} \\ &= \frac{2+3}{\sqrt{5} \cdot \sqrt{10}} \\ &= \frac{5}{\sqrt{50}} \Rightarrow \theta_1 = 45^\circ \end{aligned}$$

$$\cos \theta_2 = 90^\circ - \theta_1$$

$$\begin{aligned} \cos \theta_2 &= \frac{|\vec{CB} \cdot \vec{CA}|}{|\vec{CB}| |\vec{CA}|} \\ &= \frac{-1+6}{\sqrt{5} \cdot \sqrt{10}} \\ &= \frac{5}{\sqrt{50}} \Rightarrow \theta_2 = 45^\circ \end{aligned}$$

نقطتين في الفضاء (A)

$$\vec{a} \cdot \vec{b} = 0 \text{ لا بد ان تكون}$$

$$\vec{a} \cdot \vec{b} = (2, 1) \cdot (-3, 6)$$

$$= -6 + 6 = 0$$

مقابلان

$$\vec{c} + \vec{a} = \vec{b} \quad (P)$$

$$\begin{aligned} \vec{c} &= \vec{b} - \vec{a} \\ &= (-5, -3) - (1, -1) \\ &= (-6, -2) \end{aligned} \quad (Q)$$

$$\vec{c} - 2\vec{a} = \vec{b}$$

$$\begin{aligned} \vec{c} &= \vec{b} + 2\vec{a} \\ &= (-5, -3) - 2(1, -1) \\ &= (-5, -3) - (2, -2) \\ &= (-7, -1) \end{aligned} \quad (R)$$

$$2\vec{c} - \vec{b} = \vec{a} \quad (S)$$

$$\begin{aligned} 2\vec{c} &= \vec{a} + \vec{b} = (1, -1) + (-5, -3) \\ &= (-4, -4) \end{aligned}$$

$$\vec{c} = \frac{(-4, -4)}{2} = (-2, -2)$$

$$\begin{aligned} \vec{a} \cdot \vec{b} &= (5, -7) \cdot (3, 4) \quad (A) \quad (T) \\ &= 15 - 28 = 13 \end{aligned}$$

$$\begin{aligned} \vec{b} \cdot \vec{c} &= (3, 4) \cdot (-2, -5) \quad (Q) \\ &= -6 - 20 = -26 \end{aligned}$$

$$\begin{aligned} \vec{c} \cdot \vec{b} &= (-2, -5) \cdot (3, 4) \quad (R) \\ &= -6 - 20 = -26 \end{aligned}$$

$$\vec{AB} = (1, 4) - (3, 5) \quad (U) \quad (V)$$

$$= (-2, -1)$$

$$\vec{BC} = (2, 2) - (1, 4) = (1, -2)$$

$$\vec{AB} \cdot \vec{BC} = -2 + 2 = 0$$



$$\vec{a} \cdot \vec{b} = 2 - 3 - 4 = -5 \quad \text{Ing } \textcircled{6}$$

$$\cos \theta = \frac{-5}{\sqrt{3} \cdot \sqrt{29}} = \frac{-5}{\sqrt{87}}$$

$$\theta = \dots$$

$$\vec{a} \cdot \vec{b} = -4 - 12 + 6 = -10 \quad \text{Ing } \textcircled{7}$$

$$\cos \theta = \frac{-10}{\sqrt{29} \cdot \sqrt{17}}$$

$$\theta = \dots$$

$$(2\hat{i} + \hat{j} - \hat{k}) \cdot (3\hat{i} + 7\hat{j} + 13\hat{k}) \quad \text{Ing } \textcircled{v}$$

$$= 6 + 7 - 13 = 0$$

$$(2\hat{i} + \hat{j} - \hat{k}) \cdot (20\hat{i} - 29\hat{j} + 11\hat{k})$$

$$= 40 - 29 - 11 = 0$$

$$(3\hat{i} + 7\hat{j} + 13\hat{k}) \cdot (20\hat{i} - 29\hat{j} + 11\hat{k})$$

$$= 60 - 203 + 143 = 0$$

$$\vec{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 1 & 1 & -10 \end{vmatrix} = -11\hat{i} + \hat{j} - \hat{k} \quad \text{Ing } \textcircled{11}$$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ -1 & -3 & 4 \end{vmatrix} = 7\hat{i} + \hat{j} + \hat{k}$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = (2\hat{i} + \hat{j} - 10\hat{k}) \cdot (7\hat{i} + \hat{j} + \hat{k})$$

$$= 7 + 1 - 10 = -2 \quad \#$$

$$\vec{a} + \vec{b} = (2-1)\hat{i} + (-5+2)\hat{j} + (10-9)\hat{k}$$

$$= \hat{i} - 3\hat{j} + \hat{k}$$

$$\vec{a} - \vec{b} = 3\hat{i} - 7\hat{j} + 19\hat{k}$$

$$2\vec{a} + 3\vec{b} = (4\hat{i} - 10\hat{j} + 20\hat{k})$$

$$+ (-3\hat{i} + 6\hat{j} - 27\hat{k})$$

$$= (\hat{i} - 4\hat{j} - 7\hat{k})$$

$$\textcircled{a} \vec{a} - 2\vec{b} = -3\hat{i} - 3\hat{j} + 3\hat{k} \quad \text{Ing } \textcircled{c}$$

$$\textcircled{b} - 2\vec{a} + \vec{b} = 3\hat{j} - 3\hat{k}$$

$$\textcircled{c} 3\vec{a} + 2\vec{b} = 7\hat{i} - \hat{j} + \hat{k}$$

$$|\vec{a}| = \sqrt{9+16+1} = \sqrt{26} \quad \text{Ing } \textcircled{r}$$

$$|\vec{b}| = \sqrt{4+5+16} = \sqrt{26}$$

$$\vec{U}_a = \frac{4\hat{i} - 3\hat{j} + 12\hat{k}}{\sqrt{16+9+144}} \quad \text{Ing } \textcircled{e}$$

$$\vec{U}_a = \frac{4}{13}\hat{i} - \frac{3}{13}\hat{j} + \frac{12}{13}\hat{k}$$

$$\vec{U}_b = \frac{7\hat{i} - 3\hat{j} + \sqrt{6}\hat{k}}{\sqrt{49+9+6}}$$

$$= \frac{7}{8}\hat{i} - \frac{3}{8}\hat{j} + \frac{\sqrt{6}}{8}\hat{k}$$



$F_1 = 20, F_2 = 10$ (2)

$\theta = 120^\circ$

$$R = \sqrt{400 + 100 + 400 \cos 120}$$

$$= \sqrt{500 - 200} = 20 \text{ N}$$

$$\tan \alpha = \frac{10 \sin 120}{20 + 10 \sin 120}$$

$$= \frac{-5}{20 + 5\sqrt{3}}$$

$\alpha = \checkmark$

$R = 2f \quad c \quad 3f \quad 4f$ (3)

$$4f^2 = 9f^2 + f^2 + 6f^2 \cos \theta$$

$$4 = 9 + 1 + 6 \cos \theta$$

$$-1 = 6 \cos \theta \rightarrow \theta = 180^\circ$$

$R = \sqrt{(750)^2 + (500)^2 + 2(750)(500) \cos 60}$ (4)

$$\tan \alpha = \frac{500 \sin 60}{750 + 500 \cos 60}$$

$$\tan 45 = \frac{4 \sin 135}{F + 4 \cos 135} = 1$$

$$F = \frac{4}{\sqrt{2}} = \frac{4}{\sqrt{2}} \rightarrow F = \frac{8}{\sqrt{2}} = 4\sqrt{2}$$

القوة الناتجة عن القوة F_1 و F_2 هي 64 N

$$R = F_1 + F_2 = 22 + 42 = 64 \text{ N}$$

$F_1 = 5\sqrt{2}, F_2 = 5$ (5)

$\theta = 45^\circ$

$$R = \sqrt{50 + 25 + 50\sqrt{2} \cos 45}$$

$$= \sqrt{125}$$

$$\tan \alpha = \frac{5 \sin 45}{5\sqrt{2} + 5 \cos 45}$$

$$= \frac{5/\sqrt{2}}{5\sqrt{2} + 5/\sqrt{2}} = \frac{5}{10 + 5}$$

$$= \frac{5}{15} = \frac{1}{3}$$

$\alpha = \checkmark$

$F_1 = 10, F_2 = 10\sqrt{3}$ (6)

$R = 20 \rightarrow \theta = ??$

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \theta$$

$$400 = 100 + 300 + 200 \cos \theta$$

$$\frac{300}{200} = \cos \theta = 0$$

$\theta = 90^\circ$

$R = 20 \text{ N}$

198 (18)

$$244 = F^2 + 5F^2 + 10\sqrt{2}F^2 \cos 45$$

$$244 = 16F^2$$

$$F^2 = \frac{244}{16} \rightarrow F = \dots$$

199 (11)

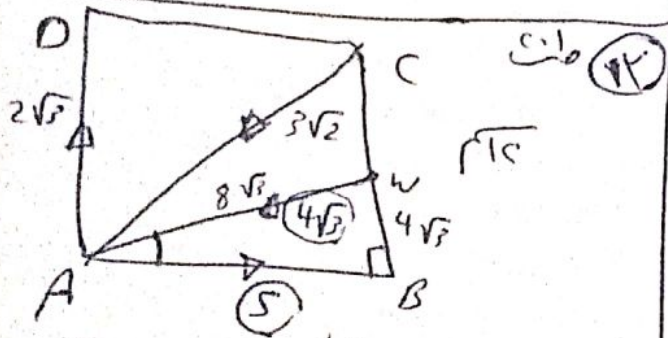
$$R_x = 3 + 5\sqrt{2} \times \frac{1}{\sqrt{2}} + 10 \times \frac{6}{10}$$

$$= 14$$

$$R_y = 5\sqrt{2} \times \frac{1}{\sqrt{2}} + 10 \times \frac{8}{10} + 1$$

$$= 14$$

$$R = \sqrt{(14)^2 + (14)^2} = 14\sqrt{2} \text{ N}$$



199 (11)

$$R_x = 5 - 4\sqrt{3} \times \frac{12}{8\sqrt{3}} - 3\sqrt{2} \cdot \frac{1}{\sqrt{2}}$$

$$= -4$$

$$R_y = -4\sqrt{3} \times \frac{4\sqrt{3}}{8\sqrt{3}} - 3\sqrt{2} \cdot \frac{1}{\sqrt{2}} + 2\sqrt{3}$$

$$= -3$$

$$R = \sqrt{9 + 16} = 5$$

$$\theta = \tan^{-1} \frac{-3}{-4} = \dots$$

190 (9)

$$\alpha = 90 \text{ m/sec}$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{\sin 90}{\cos 90} = \frac{1}{0}$$

$\rho = \dots$

$$7 + 14 \cos \theta = 0$$

$$\cos \theta = -\frac{7}{14} = -\frac{1}{2}$$

$$\theta = 240 \text{ or } 120$$

190 (10)

$$6 + F \cos 120 = 0$$

$$-\frac{F}{2} = -6 \rightarrow F = 12 \text{ N}$$

197 (11)

$$R = 10 + 15 = 25 \text{ N}$$

وغيره القوي

$$R = 15 - 10 = 5 \text{ N}$$

القوي القوي

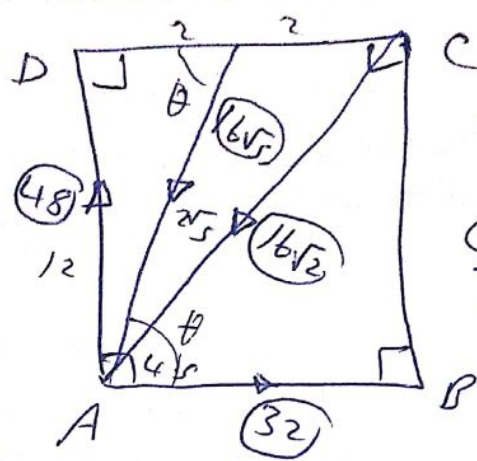
197 (10)

$$R = \sqrt{100 + 100 + 200 \cos 120}$$

$$= 10 \text{ N}$$

$$\tan \alpha = \frac{100 \sin 120}{100 + 100 \cos 120} = \frac{\frac{\sqrt{3}}{2}}{1 + \frac{1}{2}}$$

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{1}{\sqrt{3}} \rightarrow \alpha = \dots$$



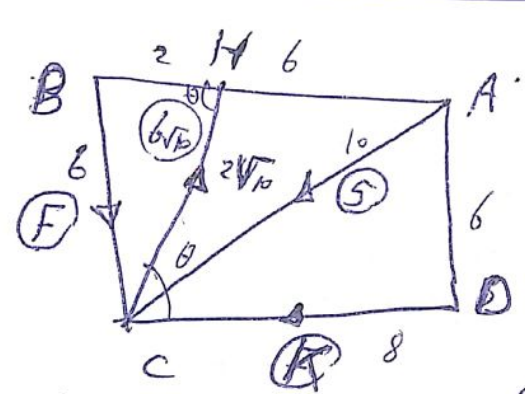
$\sum F_x = 0$
 في اتجاه x
 $\underline{\underline{32 - 16\sqrt{2}}}$
 القوة

$$R_x = 32 - 16\sqrt{2} \cdot \frac{1}{\sqrt{2}} - 16\sqrt{2} \cdot \frac{2}{2\sqrt{2}}$$

$$= 32 - 16 - 16 = 0$$

$$R_y = 48 - 16\sqrt{2} \cdot \frac{4}{2\sqrt{2}} - 16\sqrt{2} \cdot \frac{1}{\sqrt{2}}$$

$$= 48 - 32 - 16 = 0$$



$\sum F_x = 0$
 في اتجاه x
 $\underline{\underline{AH=6}}$
 $CH = 2\sqrt{5}$

$$R_x = 0 \quad -K - 5 \cdot \frac{8}{10} + 6\sqrt{5} \cdot \frac{2}{2\sqrt{5}} = 0$$

$$K + 4 - 6 = 0 \quad \rightarrow \underline{\underline{K=2}}$$

$$R_y = 0 \quad -5 \cdot \frac{6}{10} + 6\sqrt{5} \cdot \frac{6}{2\sqrt{5}} - F = 0$$

$$-3 + 18 - F = 0$$

$$\underline{\underline{F=15}}$$